

'Fault Finding and Fixing' Tasks

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WHY USE FAULT FINDING AND FIXING TASKS?

Newspapers, television and other media offer assertions and argument often based on plausible 'mathematical' reasoning. It is therefore an important life skill to be able to analyze a statement or argument critically, and to correct fallacious reasoning.

One of the most important skills of any mathematician is the ability to spot then remediate plausible errors in his or her own work and in the work of others. These tasks provide practice at this key skill.

There are a number of well-known misconceptions held by students of mathematics, many of which persist undetected into the college years. These misconceptions need to be identified and remedied, to avoid major conceptual problems, later. Many of the 'Fault finding and fixing' tasks make use of common misconceptions, and so the task set can play a diagnostic role.

WHAT ARE FAULT FINDING AND FIXING TASKS?

The tasks in this package offer students a number of mathematical mistakes which they are asked to diagnose and rectify. These require students to analyze mathematical statements and deduce from the context the part that is most likely to contain the error (there may be more than one possibility), explain the cause of the error and rectify it. Such tasks can be quite demanding. It is often more difficult to explain the cause of another's seductive error than to avoid making it oneself. Contexts include: percentages; graphical interpretation; and reasoning from statistical data.

WHAT IS INVOLVED?

Instructor Preparation Time:	Minimal if use existing tasks.
Preparing Your Students:	Students will need some coaching on their first task.
Class Time:	Some tasks take 5 mins; others as much as 45 mins. Tasks can be assembled in a number of combinations.
Disciplines:	Appropriate for all, requires proportional reasoning and graphical skills.
Class Size:	Any.
Special Classroom/Technical Requirements:	None.
Individual or Group Involvement:	Either.
Analyzing Results:	Intensive for formal scoring for large

	classes. Best used as an informal way to get your students thinking mathematically. Useful for identifying common student misconceptions.
Other Things to Consider:	None.s.

Description

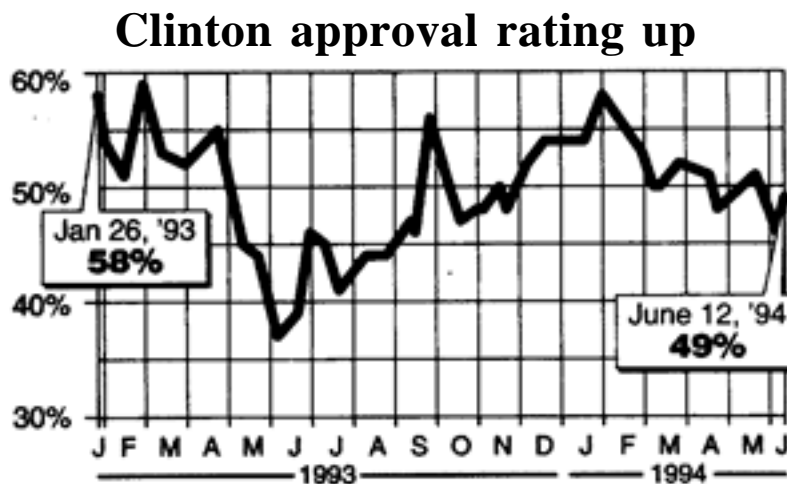
A collection of situations, graphs, diagrams and charts is presented that have already been interpreted by someone (in some cases in a newspaper). These interpretations are usually flawed. The students' task is to critique the interpretation or representation and say how it may be improved. The task collection begins with elementary (though common) mistakes involving percentages, then presents problems involving combinations and chance. Some tasks require graphical interpretation, and others require the interpretation of statistical data. Many of the tasks are taken from everyday experiences, and mistakes in published materials such as newspapers and books.

The mistakes are ones that are made commonly, and it is likely that students in some classes will endorse the misconceptions exposed in the tasks.

Example of "Fault Finding and Fixing" Task

Presidential Popularity

The following headline and chart appeared in the June 14, 1994 issue of *USA Today* newspaper.



Source: USA TODAY/CNN/Gallup Poll of 756 adults by telephone on June 11–12. Margin of error: ± 4 percentage points.

With D-Day observances over and President Clinton back home, voters' attitudes toward the president are settling down a bit. Now that attention is back on the economy, health care and crises in Bosnia and Haiti, a USA Today / CNN / Gallup Poll taken over the weekend [of June 12] shows Clinton's job performance rating inching upward to 49% ... It's an improvement from a poll taken [on June 6] as Clinton was in Europe marking the 50th anniversary of the Allied invasion of Normandy, which showed approval dropping to 46% ...

Write a letter to the editor of *USA Today* explaining why the assertion "Clinton

approval rating up” might be regarded as questionable or misleading. Be concise (editors prefer letters that are brief and to the point) but convincing (give mathematical evidence to support your claims).

Assessment purposes

- To see if students are able to identify and explain the cause of common mathematical mistakes;
- To see if students can rectify the mistakes;
- To identify student misconceptions.

Limitations

Little mathematical knowledge is assumed apart from fundamental ideas around percentages, graphical interpretation and data display, and chance and proportion. However, students are required to understand the concepts, not simply to demonstrate competence in using routine procedures, and must use their knowledge to make sense of situations.

Teaching Goals

- Students learn to analyze a statement or argument critically, and to correct fallacious reasoning.
- There is some scope for the diagnosis and remediation of student misconceptions in some fundamental aspects of mathematics.

Suggestions for Use

Introducing 'Fault finding and fixing' tasks for the first time.

You might choose to introduce 'Fault finding and fixing' as a core element of mathematical thinking. Identifying and fixing mistakes early makes problem solving in mathematics far easier! The relevance of 'Fault finding and fixing' skills to everyday life is immediately obvious, and provides a clear justification for the use of these materials.

Providing guidance as students work on 'Fault finding and fixing' tasks.

Research evidence shows that correcting student misconceptions is enhanced by 'cognitive conflict'. This means that situations are established where the student is made aware of the existence of the gap between what they believe to be true, and the actual situation. Group work can provide a good source of this conflict, if students disagree about the truth or otherwise of the statements being made. As a teacher, you might choose to provoke the cognitive conflict as much as possible, and avoid giving correct answers. Students who struggle to disentangle their misconceptions are likely to learn more than students who are simply told the correct answer to the particular problem they face. Asking for student explanations as part of class review can be powerful; it is worth identifying students who are likely to offer conflicting views.

Reporting out of individual or group work

If you decide to organize a whole group discussion on what students came up with, it is useful to decide the degree to which you will participate in these discussions. You can facilitate the students' discussion, having them defend their ideas and write their ideas on the board, while adding almost none of your own. During discussions, you are likely to pose further questions that provoke debate. At some point, you are likely to want to review the answers to individual tasks,

and to emphasize the need for fault finding and fixing as a key component of mathematical thinking.

Formal and informal use of 'Fault finding and fixing' tasks

There is a considerable variety of tasks in this task set, and the time required on each task ranges from just a few minutes to a whole class period. Tasks can be used formally - by assembling tests from the task collection - or informally, by dropping a 'Fault finding and fixing' task into your regular instruction, for example. In formal assessment (where you grade the assignment as an examination), do not intervene except where specified. Even modest interventions – reinterpreting instructions, suggesting ways to begin, offering prompts when students appear to be stuck – have the potential to alter the task for the student significantly.

In informal assessment (an exercise, graded or non-graded), you may want to be less rigid in giving the students help. Under these circumstances, you may reasonably decide to do some coaching, talk with students as they work on the task, or pose questions when they seem to get stuck. In these instances you may be using the tasks for informal assessments—observing what strategies students favor, what kinds of questions they ask, what they seem to understand and what they are struggling with, and what kinds of prompts get them unstuck. This can be extremely useful information in helping you make ongoing instructional and assessment decisions.

Group work versus individual work

These tasks are appropriate for group work. Students can discuss the claims made, and disagreements are likely to lead to enhanced learning. The CL1 collaborative learning site can provide instructions on how to use group work effectively within the classroom. Conversely, an analysis of individual work may give you more clues as to misconceptions held by individual students.

Presumed background knowledge

Little mathematical knowledge is assumed apart from fundamental ideas around percentages, graphical interpretation and data display, and chance and proportion.

Step-by-Step instructions

1. Prepare by reading through the 'Fault finding and fixing' tasks on your own and coming up with your own solutions.
2. Hand out copies of the task to students, either working individually or in groups.
3. State the your goals for the 'Fault finding and fixing' task, emphasizing that they should be able to defend both their choice of method and the reasoning which leads to their answer.
4. Walk around and listen to students as they discuss and work through the problems, providing guidance as necessary.
5. Have students present their solutions, either in written or verbal form.

Variations

The tasks included in this site can be downloaded and used without modification. If you choose to develop your own 'Fault finding and fixing' task, using information in newspapers, for example, you can follow the pattern used here.

For more formal investigations of student misconceptions, purpose-built assessments are available.

Analysis

Solutions are given for each of the tasks in the task set. Student performance can be judged against the answers provided.

Examination of performance across a range of tasks will identify students with weak conceptual knowledge in a number of areas of mathematics, and students with poor skills in faultfinding and fixing.

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Most assessment practices seem to emphasise the reproduction of imitative, standardised techniques. I want something different for my students. I want them to become mathematicians - not rehearse and reproduce bits of mathematics.

I use the five 'mathematical thinking' tasks to stimulate discussion between students. They share solutions, argue in more logical, reasoned ways and begin to see mathematics as a powerful, creative subject to which they can contribute. Its much more fun to try to think and reach solutions collaboratively. Assessment doesn't have to be an isolated, threatening business.

Not just answers, but approaches.

Malcolm Swan is a lecturer in Mathematics Education at University of Nottingham and is a leading designer on the MARS team. His research interests lie in the design of teaching and assessment. He has worked for many years on research and development projects concerning diagnostic teaching (including ways of using misconceptions to promote long term learning), reflection and metacognition and the assessment of problem solving. For five years he was Chief Examiner for one of the largest examination boards in England. He is also interested in teacher development and has produced many courses and resources for the inservice training of teachers.

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Thinking mathematically is about developing habits of mind that are always there when you need them - not in a book you can look up later.

For me, a big part of education is about helping students develop uncommon common sense. I want students to develop ways of thinking that cross boundaries - between courses, and between mathematics and daily life.

People should be able to tackle new problems with some confidence - not with a sinking feeling 'we didn't do that yet'. I wanted to share a range of big ideas concerned with understanding complex situations, reasoning from evidence, and judging the likely success of possible solutions before they were tried out. One problem I had is that my students seemed to learn things in 'boxes' that were only opened at exam time. Thinking mathematically is about developing habits of mind that are always there when you need them - not in a book you can look up later.

You can tell the teaching is working when mathematical thinking becomes part of everyday thinking. Sometimes it is evidence that the ideas have become part of the mental toolkit used in class - 'lets do a Fermi [make a plausible estimate] on it'. Sometimes it comes out as an anecdote. On graduate told me a story of how my course got him into trouble. He was talking with a senior clinician about the incidence of a problem in child development, and the need to employ more psychologists to address it. He 'did a Fermi' on the number of cases (wildly overestimated) and the resource implications (impossible in the circumstances). He said there was a silence in the group...you just don't teach the boss how to suck eggs, even when he isn't very good at it. He laughed.

Jim Ridgway is Professor of Education at the University of Durham, and leads the MARS team there. Jim's background is in applied cognitive psychology. As well as kindergarten to college level one assessment, his interests include the uses of computers in schools, fostering and testing higher order skills, and the study of change. His work on assessment is diverse, and includes, the selection of fast jet pilots, and cognitive analyses of the processes of task design. In MARS hhe has special responsibility for data analysis and psychometric issues, and for the CL-1 work.

About MARS

The Mathematics Assessment Resource Service, MARS, offers a range of services and materials in support of the implementation of balanced performance assessment in mathematics across the age range K to CL-1. MARS is funded by the US National Science Foundation, and builds on earlier funding which began in 1992 for the Balanced Assessment Project (BA) from which MARS grew.

MARS offers effective support in:

The Design of Assessment Systems: assessment systems are tailored to the needs of specific clients. Design ranges from the contribution of individual tasks, through to full scale collaborative work on test development, scoring and reporting. Clients include Cities, States, and groups concerned with educational effectiveness, such as curriculum projects and professional development initiatives.

Professional Development for Teachers: most teachers need help in preparing their students for the much wider range of task types that balanced performance assessment involves. MARS offers professional development workshops for district leadership and 'mentor teachers', built on materials that are effective when used later by such leaders with their colleagues in school.

Developing Design Skills: many clients have good reasons to develop their own assessment, either for individual student assessment or for system monitoring. Doing this well is a

challenge. MARS works with design teams in both design consultancy and the further development of the team's own design skills.

To support its design team, MARS has developed a database, now with around 1000 interesting tasks across the age range, on which designers can draw, modify or build, to fit any particular design challenge.