Always, Sometimes or Never True - Set #1 (solutions)

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Introduction:

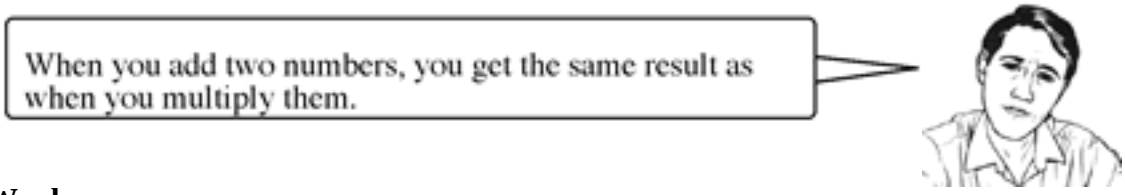
You will be given a number of statements. You must decide if each statement is

- always true, or
- sometimes true, or
- never true

You must provide full and convincing reasons for your decision. If you think that a statement is sometimes true, you must fully explain *when* it is true and *when* it is not true.

Here is an example of what we mean:

Example:



Weaker response:

This statement is sometimes true.
It is true when both numbers are 0 and when both numbers are 2.
It is not true when one number is 2 and one number is 3.

Stronger response:

This statement is sometimes true.
Suppose one number is x and one number is y.
The statement says that: $x+y = xy$
This simplifies to the condition that $y = x/(x-1)$

A few pairs of numbers when it works are therefore:
(0, 0); (2, 2); (3, 3/2); (4, 4/3); (5, 5/4) .....

There are also other pairs which work!

The aim of this assessment is to provide the opportunity for you to:

- test statements to see how far they are true;
- provide examples or counterexamples to support your conclusions
- provide convincing arguments or proofs to support your conclusions

For each statement, say whether it is always, sometimes or never true.
You must provide **several examples or counterexamples** to support your decision.
Try also to provide **convincing** reasons for your decision.
You may even be able to provide a **proof** in some cases.

1. The more digits a number has, then the larger is its value.

Is this always, sometimes or never true? .....

Reasons or examples:

Sample Solution: Sometimes true.
This statement is true when we are dealing with positive integers only.
For negative integers, such as -23 and -234, the more digits, the smaller the value of the number.
For decimals, the number of digits tells you nothing about the size of the number (*e.g.*, $0.62 > 0.236$ but $0.12 < 0.236$).

2. If you multiply 12 by a number, the answer will be greater than 12.

Is this always, sometimes or never true? .....

Reasons or examples:

Sample Solution: Sometimes true.
$12x > 12$ only when $x > 1$.

3. The square of a number is greater than that number.

Is this always, sometimes or never true? .....

Reasons or examples:

Sample Solution: Sometimes true.
$x^2 > x$ when $x(x - 1) > 0$. That is only when $x < 0$ or $x > 1$.

4. If two rectangles have the same perimeter, they have the same area.

Is this always, sometimes or never true? .....

Reasons or examples:

Sample Solution: Sometimes true.
This is only true when the rectangles are identical.

5. Pentagons have fewer right angles than rectangles.

Is this always, sometimes or never true? .....

Reasons or examples:

Sample Solution: Always true.
Pentagons can have at most 3 right angles. Rectangles must have four.
If one tries to draw a five-sided polygon with four or more right angles, then it either degenerates into a rectangle, or has three parallel sides and thus cannot be closed.

6. Quadrilaterals tessellate.

Is this always, sometimes or never true? .....

Reasons or examples:

Sample Solution: Always true.
This follows from the property that the four angles of a quadrilateral total 360˚.