Always, Sometimes or Never True - Set #2 (solutions)

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Introduction:

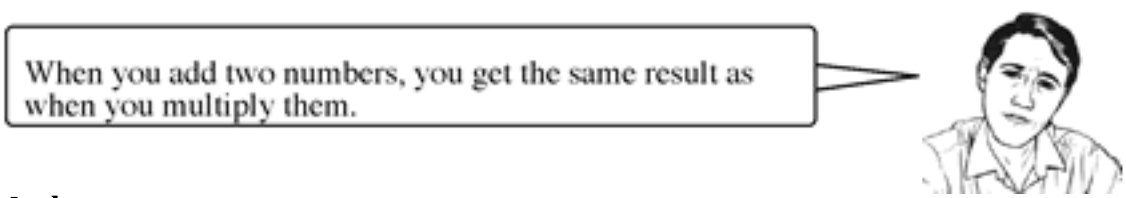
You will be given a number of statements. You must decide if each statement is

- always true, or
- sometimes true, or
- never true

You must provide full and convincing reasons for your decision. If you think that a statement is sometimes true, you must fully explain *when* it is true and *when* it is not true.

Here is an example of what we mean:

Example:



Weaker response:

This statement is sometimes true.
It is true when both numbers are 0 and when both numbers are 2.
It is not true when one number is 2 and one number is 3.

Stronger response:

This statement is sometimes true.
Suppose one number is x and one number is y.
The statement says that: $x+y = xy$
This simplifies to the condition that $y = x/(x-1)$

A few pairs of numbers when it works are therefore:
(0, 0); (2, 2); (3, 3/2); (4, 4/3); (5, 5/4) ......

There are also other pairs which work!

The aim of this assessment is to provide the opportunity for you to:

- test statements to see how far they are true;
- provide examples or counterexamples to support your conclusions
- provide convincing arguments or proofs to support your conclusions

For each statement, say whether it is always, sometimes or never true.
You must provide **several examples or counterexamples** to support your decision.
Try also to provide **convincing** reasons for your decision.
You may even be able to provide a **proof** in some cases.

1. The center of a circle that circumscribes a triangle is inside the triangle.

Is this always, sometimes or never true? ....

Reasons or examples:

Sample Solution: Sometimes true.
The center of the circle that circumscribes a triangle *sometimes* lies inside the triangle. The statement is true when the triangle is acute. In the case of a right triangle the center of the circumscribing circle lies *on* the triangle. In the case of an obtuse triangle, the center of the circumscribing triangle lies outside the triangle.

2. An altitude subdivides a triangle into two similar triangles.

Is this always, sometimes or never true? ....

Reasons or examples:

Sample Solution: Sometimes true.
It is *sometimes* true that an altitude subdivides a triangle into similar triangle. Two interesting case are that of an altitude drawn in an isosceles triangle from the non-base angle, and that of an altitude drawn in a right triangle from the right angle.

3. $(a + b)^2 = a^2 + b^2$

Is this always, sometimes or never true? ....

Reasons or examples:

Sample Solution: Sometimes true.
This is only true when $a = 0$ or $b = 0$ or $a = b = 0$.

4. $3x^2 = (3x)^2$

Is this always, sometimes or never true? ....

Reasons or examples:

Sample Solution: Sometimes true.
Again this is true only when $x = 0$

5. A shape with a finite area has a finite perimeter.

Is this always, sometimes or never true? ....

Reasons or examples:

Sample Solution: Sometimes true.

If the shape has fixed area A and dimensions x and $^A/_x$. The perimeter is thus $2(x + {}^A/_x)$. As x is varied, this can be made as large or as small as we please. So theoretically, the perimeter may become infinite - but then is an infinitely thin rectangle still a rectangle (it feels more like a line to me!)? As an aside, it may also be noted that many fractals also have infinite boundaries but enclose finite areas. Moving up a dimension, it is also interesting to note that the trumpet shape obtained by rotating the curve $y = {}^1/_x$ around the x axis for $x > 1$ has a finite volume, but an infinite surface area. (Does this suggest that we can fill the shape with a finite amount of paint, but that the paint will never fully coat the surface?)

6. A shape with a finite perimeter has a finite area.

Is this always, sometimes or never true? ....

Reasons or examples:

Sample Solution: Always true.

It is always true that a shape with a finite perimeter has a finite area. If the dimensions of the rectangle are x and y, then if $2(x + y)$ is finite and x and y are both positive, both x and y must be finite, so the area must be finite.