'Creating Measures' Awkward-ness Task - Example #5 (solutions)

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This problem gives you the chance to:

- invent your own measure for the concept of "awkward-ness"
- use your measure to put situations in order of "awkward-ness"
- generalize your measure to work in different situations.

- Have you ever arrived at a packed theater after the show has started?
- You have to make everyone stand while you squeeze past to take your seat.
- Imagine that five people A, B, C, D and E each arrive to take their seat in a theater.
- They are not allowed to take different seats to the one they have been allocated.

This diagram shows the order in which they arrive and their seating positions:

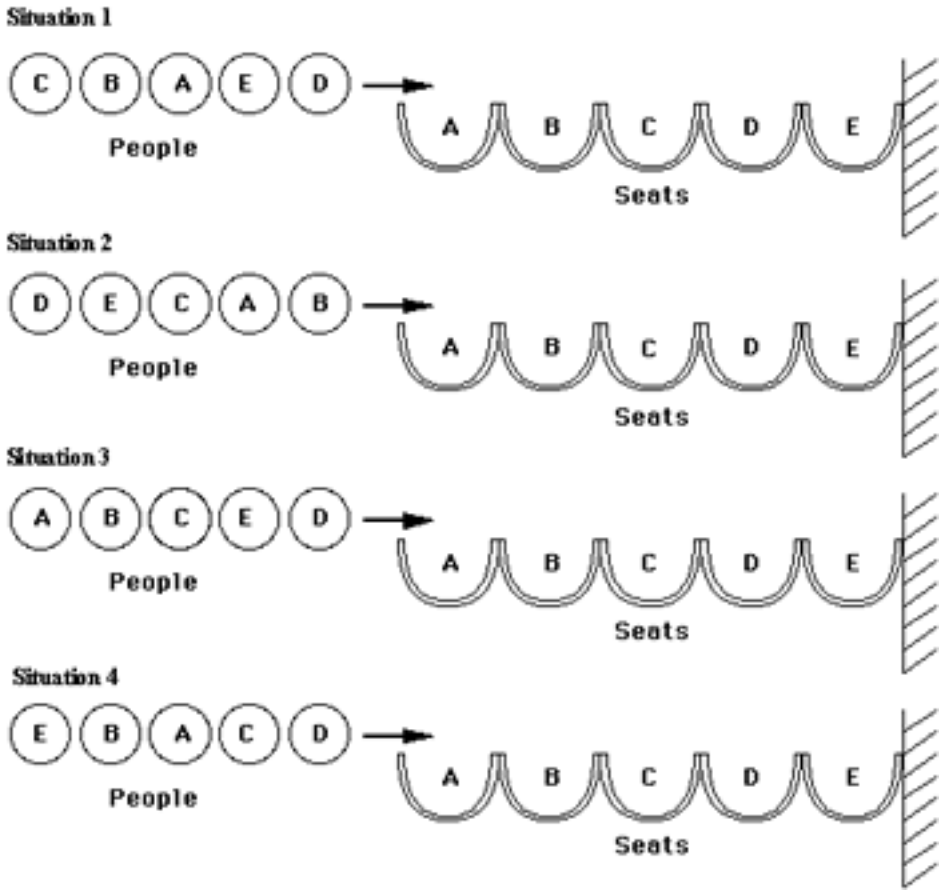
- So, D arrives first and sits in the second seat from the right hand end of the row.
- Then E arrives. D has to stand up while E squeezes into the last seat in the row.
- Then A arrives. She sits on the first seat of the row.
- Now B arrives and makes A stand, while he takes the second seat in.
- Finally C arrives and makes both A and B stand up while she takes her seat.

Warm-up

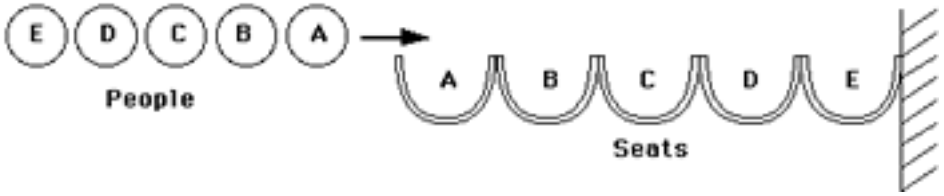
Try out this situation from different starting points using scraps of paper labeled A, B C, D and E until you can see what is happening.
What is the most awkward situation you can devise?

Draw it below:

Here are four movie theater situations:



Comment:
The most awkard situation possible is shown below:



In this situation, A sits first, then

- A stands while B takes her place
- A and B stand while C takes his place
- A, B and C stand while D takes his place
- A, B, C and D stand while E takes her place.

1. Place the four situations in order of "awkward-ness."

- Which is the easiest situation for people?
- Which is the most awkward?
- Explain how you decided.

Solution:
The above measure is unsatisfactory because:

The easiest situation is situation (3), because this results in only one person having to stand on one occasion (person D has to stand while E squeezes by).

The most awkward situation is probably (4) because people have to stand on five occasions. (A has to stand while B sits down, then A, B, C and D all have to stand while E sits down.)

2. Invent a way of measuring "awkward-ness." This should give a number to each situation. Explain carefully how your method works.

Solution for Questions 2 and 3:
A suitable measure of "awkward-ness" would be to count the number of times a person makes someone stand up to let them pass. This would give, for situations 1 to 4:

	Number of times person makes someone else stand					
Situation	**A**	**B**	**C**	**D**	**E**	**Total**
1	0	1	2	0	1	4
2	0	0	2	3	3	8
3	0	0	0	0	1	1
4	0	1	0	0	4	5

Using the totals, we have, from least to most awkward:
Situations 3, 1, 4 then 2.

3. Show how you can use your measure to place the four situations in order of "awkward-ness." Show all your work.

4. Adapt your measure so that the minimum value it can take is 0 (where no-one is made to stand up) and the maximum it can take is 1 (the most awkard situation possible).

Solution:
To make the measure range from 0 to 1, we could divide the totals above by the maximum possible "awkward-ness" score for five people = 10 (see Warm-up).

5. Show how your measure in part 4 may be generalised for any number of people entering a row. (That is when *n* people enter a row with *n* available seats).

Solution:

If there was just one person, the maximum "awkward-ness" = 0.
For 2 people, the maximum "awkward-ness" = 1.
For 3 people, the maximum "awkward-ness" = 3 (= 1+2).
For 4 people, the maximum "awkward-ness" = 6 (= 1+2+3).
For 5 people, the maximum "awkward-ness" = 10 (= 1+2+3+4).
…

For *n* people, the maximum "awkward-ness" = $\frac{n(n-1)}{2}$ (= 1 + 2 + 3 + ... n).

Thus, if s = The number of occasions on which people have to stand;
we can define our measure of "awkward-ness" for a given situation to be:

$$s \div \left(\frac{n(n-1)}{2}\right)$$

$$= \frac{2s}{n(n-1)}$$