

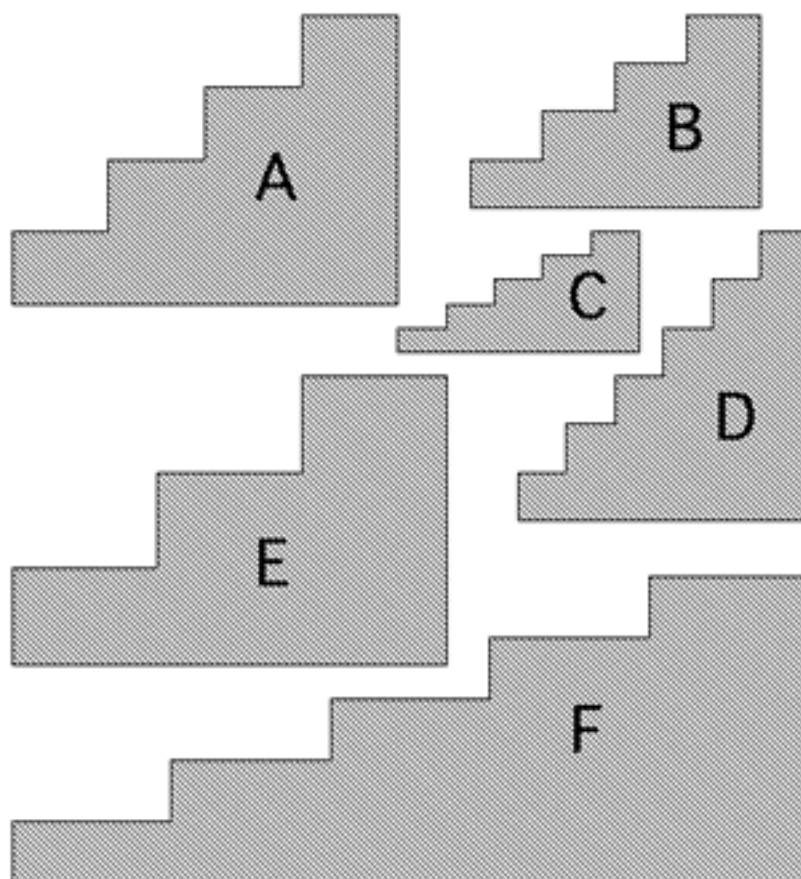
'Creating Measures' Steep-ness Task - Example #2 (solutions)

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This problem gives you the chance to:

- criticise a given measure for the concept of "steep-ness"
- invent your own ways of measuring this concept
- examine the advantages and disadvantages of different methods.



Warm-up

Without measuring anything, put the above staircases in order of "steep-ness."

Comment:

This first question is simply intended to orientate the students to the task. It may be used as a class introduction.

1. Someone has suggested that a good measure of "steep-ness" is to calculate the difference:

Height of step - length of step

for each staircase. Use this definition to put the staircases in order of "steep-ness."
Show all your work.

Solution

Using the measure '**height of each step - length of each step**', the 'steep-ness' of each staircase is given in the table below (using centimeters as the unit).

Staircase	A	B	C	D	E	F
Height (cm)	1.5	1	0.5	1	2	1.25
Length (cm)	2	1.5	1	1	3	3.33
Height-Length (cm)	-0.5	-0.5	-0.5	0	-1	-2.08

Using this measure, the staircases in order from most to least steep are:

D, A and B and C (tie), E, F.

2. Using your results, give reasons why **Height of step - length of step** is not a suitable measure for "steep-ness."

Solution:

The above measure is unsatisfactory because:

- It gives no real indication of the steepness. Using this measure, A and C are labeled as equally steep, which does not fit with intuition.
- It is dependent on the units used. If we use inches instead of centimetres we get a different "steep-ness" measure.
- It is usually negative, which is inelegant and awkward to use.

3. Invent a better way of measuring "steep-ness." Describe your method carefully below:

Solution:

There are many other ways of measuring "steep-ness." Students might, for example, propose using:

- a) The angle of inclination;
- b) The ratio of 'step height'/'step length' (technically: riser/run);
- c) The ratio of 'height of whole staircase'/'length of whole staircase';

These are equally sensible, and equivalent, except it may be sometimes unclear what we measure as the 'length' of the staircase.

4. Place the staircases in order of "steep-ness" using your method. Show all your work.

Solution:

Whichever measure we now use (a), (b) or (c), we obtain the same order for the staircases.

Staircase	A	B	C	D	E	F
Height (cm)	1.5	1	0.5	1	2	1.25
Length (cm)	2	1.5	1	1	3	3.33
Height Length (2 d.p.)	0.75 (3/4)	0.67 (2/3)	0.5 (1/2)	1 (1/1)	0.67 (2/3)	0.38 (3/8)
Angle of inclination (nearest degree)	37°	34°	27°	45°	34°	21°

This gives the order of steep-ness (from most to least steep) as:

D, A, B and E (tie), C and F.

5. Do you think your measure is a good way of measuring "steep-ness?" Explain your reasoning carefully.

Solution:

Here we would like students to review their results critically and decide whether the results from their measurements accord with their intuitions.

6. Describe a different way of measuring "steep-ness."
Compare the two methods you invented. Which is best? Why?

Solution:

This question provides an opportunity for students to look for an alternative measure.